

A THERMO-VISCOPLASTIC CONSTITUTIVE MODEL FOR CLAYS

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SUMMARY

The effect of heat on clay behaviour is characterized by non-linearity and irreversibility. Due to the complex influence of temperature, thermomechanical factors have to be taken into account for the numerical simulation of the behaviour of such materials. A cyclic thermo-viscoplastic model is developed for this purpose. It includes thermal hardening and the evolution of yield surfaces with temperature. From the physical point of view, it is built on the basis of available experimental results for a temperature range in which no phase change occurs. Conceptually, it is the generalization of an isothermal multimechanism cyclic model. A thermoplastic formulation of the model is also derived. The results obtained from numerical simulations compare well with experiments. © 1997 by John Wiley & Sons, Ltd.

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(No. of Figures: 21 No. of Tables: 2 No. of Refs: 27)

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1. INTRODUCTION

Temperature strongly affects the behaviour of clays through its influence on free and adsorbed water. Strain is caused in clay structure mainly by the thermal expansion of the free water and the loss of adsorption forces in the adsorbed water. Due to thermomechanical deformation of the solid skeleton, related to thermoelastic dilation as well as irreversible thermal strain, the hydraulic gradient and transitory flow of pore water are also modified.

There are numerous experimental results available in the literature that show the influence of temperature on the behaviour of soils.^{1–4} We notice that there are some differences in these results. Despax², for example, reports a reduction in the slope of the critical-state line in the deviatoric–isotropic (p – q) stress plane for a temperature increase. Further he observed that the change of this slope is independent of the overconsolidation ratio. Hueckel and Baldi⁵ claim that the strength appears to be mildly affected by temperature: decreasing for overconsolidated clay and possibly increasing for normally consolidated clay. These differences may be attributed to the complexity in the nature of different clays. It is reasonable to expect that the structure and sensitivity of different clay mineral components will influence their thermomechanical behaviour in different ways (e.g. montmorillonite versus kaolinite).

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It has been observed that the variation of temperature can also affect significantly the dilatancy/contractance characteristics of soils. In the case of clays for which the intrinsic viscosity is an important parameter, the physical phenomena become more complicated, as during creep the strain rate increases with the temperature.

The study of the effects of heat on clays necessitates suitable mathematical models for studying the thermo-hydro-mechanical phenomena,⁶ and in particular a constitutive model taking into account the thermomechanical behaviour of the solid skeleton.^{3,7-9} Sujit and Senol⁸ presented a formulation in which the thermal effects are taken into account in an elastoplastic model with a von Mises yield surface.

Hueckel and Borsetto⁹ proposed an extension of the elastoplastic modified Cam-Clay model, in which the thermal effects are included following Prager's thermoplasticity theory. This model is one of the rare constitutive models in this domain for which a quantitative validation has been attempted.⁵ Thermal effects are introduced at two levels: reversible dilation is added to the elastic component of the strain tensor and a state variable is introduced in the expression of the yield surface. The dependence on temperature is directly connected to consolidation pressure. The yield surface is composed of two complementary surfaces. Overconsolidated and normally consolidated conditions are separately analysed; the former are handled by thermoelasticity while the later are treated by thermoplasticity. Thermoplasticity results in a shrinking of the yield surface in the stress space with increasing temperature. In that way, it is possible to simulate, for example, the failure of clay due to thermal heating under undrained conditions.

In this paper we present a cyclic, thermoviscoplastic model in which we only consider the temperature range in which no phase change occurs. It is a generalization of the cyclic, multimechanism viscoplastic model presented by Aubry *et al.*¹⁰ We will also show that an alternative rate-independent model may be considered. Then the obtained model, when applicable, requires less parameters than the thermoviscoplastic one.

The work presented in this paper can be distinguished from others' works since:

- (1) to the authors' knowledge, it is one of the first cyclic thermoviscoplastic constitutive models able to represent principal thermomechanical characteristics of clays in a wide range of loading conditions,
- (2) it handles overconsolidated as well as normally consolidated clays in the same framework,
- (3) thermoplastic formulation, when applicable, is robust and needs less model parameters but results in heavier computations,
- (4) the isotropic thermoplastic mechanism appears to be the major element in the model. Other Cam-Clay-family models may be transformed to thermoplastic models by coupling such a mechanism with their standard yield surface.

In the following sections we will distinguish three types of loading paths: isothermal-mechanical, thermal and thermomechanical. An isothermal-mechanical loading path corresponds to standard mechanical loading at a given temperature and the response will be presented in the strain/stress plane (Figure 1). A thermal loading path represents the response of soil to thermal loading (difference between present temperature and initial or reference temperature) at constant stress and results will be given in the strain/temperature plane (Figure 2). Finally, a coupled thermomechanical loading path includes both thermal and mechanical loading. The thermal and mechanical loading increments are not imposed simultaneously in order to permit the distinction between their effects (Figure 3).

For all cases both drained and undrained conditions are considered here. In drained tests used, the loading increments are slow enough to assume a complete drainage of the pore pressure.

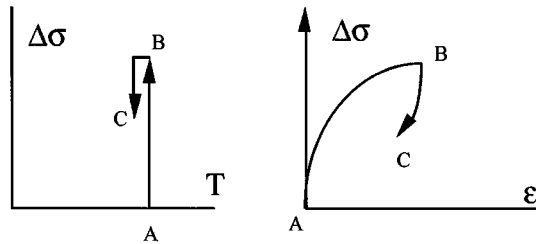


Figure 1. Isothermal-mechanical loading path

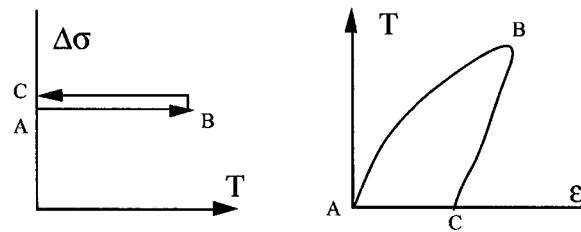


Figure 2. Thermal loading path

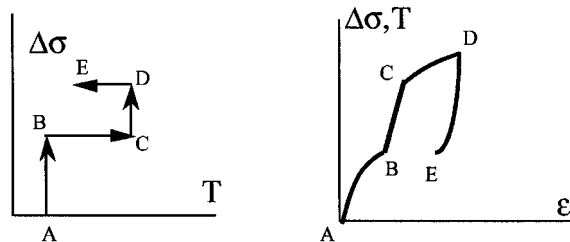


Figure 3. Thermomechanical loading path

In following sections we present a brief state of art on thermomechanical behaviour of clays. Then, we introduce a constitutive thermomechanical model capable to simulate the major aspects of clay behaviour under different coupled thermomechanical loading paths. The model's response is discussed and validation tests are carried out in the last section.

2. THERMOMECHANICAL BEHAVIOUR OF CLAYS

In this section we list the principal consequences of heat variation on the effective behaviour of clays. The thermomechanical response of clays mostly depends on their physical and chemical characteristics (mineralogical nature, interparticle forces, hydration, ions and consistency index). For example, the development of double layer is conditioned by the nature of adsorbed ions and the interparticle forces that govern internal friction and mechanical resistance¹¹ depend on this double layer.

In general, when a clay sample is heated slowly enough to allow a complete draining, two phenomena take place.^{1, 11–14}

- (i) a reversible one, so-called thermoelasticity, due to the dilation of the mineral components that is often assumed to be independent from the stress state,
- (ii) an irreversible one, so-called thermoplasticity, caused both by the dilation of clay minerals and collapse of adsorbed water, resulting in failure of some interparticle ties and the consequent rearrangement of the internal structure of the soil skeleton. The suppression of these ties corresponds to the elimination of the equilibrium between attractive and repulsive forces, and under constant compressive stress causes a volume reduction of the sample (see Figure 4 for oedometric and Figure 5 for isotropic conditions).

Two different thermomechanical responses may be distinguished in isotropic or deviatoric stress-loading conditions, as follows.

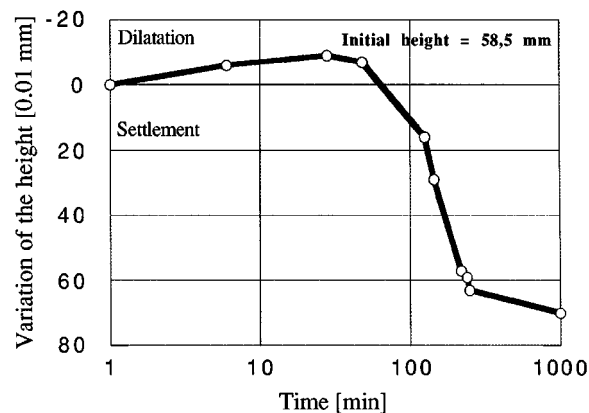


Figure 4. Variation of the height of Vallauris Clay during its heating from 22 to 58°C at constant stress in oedometric conditions; initial vertical stress 5 kPa (from Fleureau¹¹)

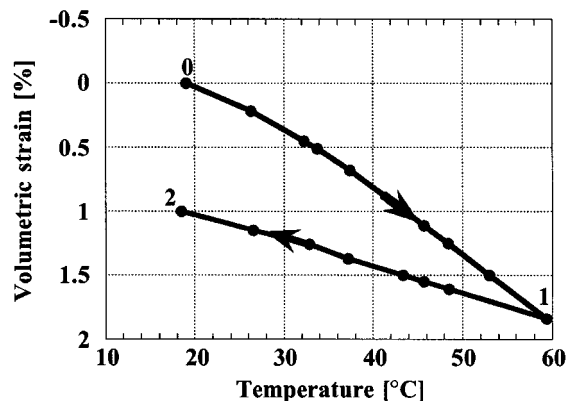


Figure 5. Effects of temperature variation on a saturated illite sample under constant isotropic compression (cell pressure 400 kPa, back pressure 200 kPa) (from Campanella and Mitchell¹)

Isotropic stress conditions

Figure 5 shows the response of a sample of saturated drained illite to a thermal loading–unloading cycle at constant isotropic stress. We note that the sample settles during the temperature increase (points 0 to 1). The volume variation is non-linear. During the cooling phase, on the contrary, a relatively linear behaviour resulting in the volume increase is observed (points 1 to 2). The whole cycle indicates the irreversibility of strain due to thermal loading–unloading. Such permanent strain in a cycle of thermal loading is representative of thermal hardening.

As noted before, the rupture of interparticle ties results in a higher contraction for samples. This phenomenon was illustrated by Plum and Esrig¹³ where the influence of the overconsolidation ratio on the response to thermal loading of illite is shown (Figure 6). We notice that for a given rise in temperature, the compaction of a soil volume is smaller for higher overconsolidation ratios.

Experimental work^{2,11–13} also shows that in the $(e - \ln p)$ plane, with e being the void ratio and p being the mean effective pressure, the slope of the consolidation line is independent of temperature. The experiments on saturated illite¹ at three different temperatures (Figure 7) show also that heating applied prior to loading produces a densification of the sample at constant isotropic pressure.

Deviatoric stress conditions

In triaxial tests, a loss of deviatoric resistance was observed in heated illite clay specimens.¹³ Figure 8 shows the variation of the internal friction angle at critical state in black clay at different temperatures (from 22 to 88°C) in unconsolidated undrained triaxial tests. Mitchell *et al.*⁷ have found the same results that indicate that strength of clay reduces with temperature and used the theory of kinetic processes to explain it. The dependence of the deviatoric response on heating is twofold: When normally consolidated drained samples are subject to thermal loading before the shear tests, the shear resistance increases.¹⁶ This may be explained by the fact that in normally consolidated clays thermal compaction takes place and density increases. On the other hand, for

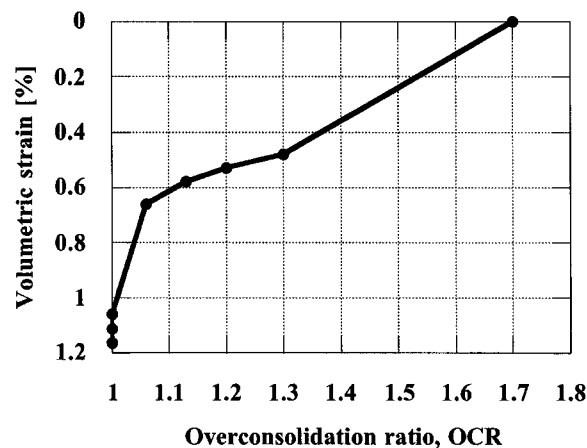


Figure 6. Influence of overconsolidation ratio on the deformation of illite heated from 24 to 50°C (from Plum and Esrig¹³)

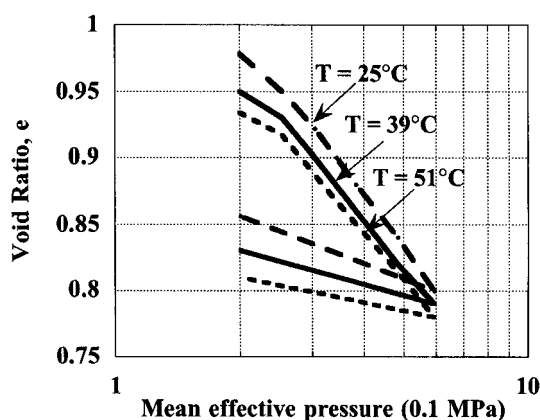


Figure 7. Isotropic consolidation of illite at three different temperatures (from Campanella and Mitchell¹⁴)

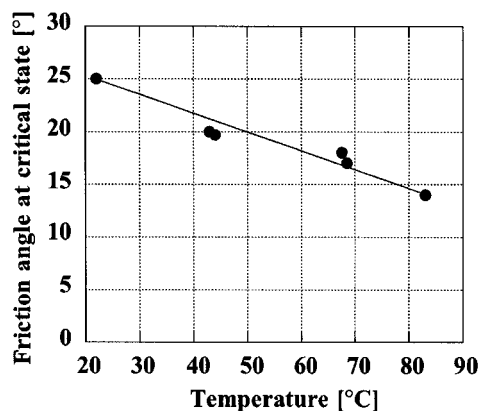


Figure 8. Variation of deviatoric resistance with temperature (from Hicher¹⁵)

a similar initial density, a higher temperature may provoke a smaller shear resistance^{14,15} (depending on the clay's nature) due to the decrease of the friction angle at the critical state.

Summarizing, the experimental work shows that the temperature variation has two major effects on clays:

- (a) a thermal reversible dilation as well as an irreversible contracting strain. This last one is dominant in the case of a small overconsolidation ratio;
- (b) modification of mechanical behaviour; quantitatively, this corresponds to a continuous variation of mechanical characteristics (e.g. stiffness, strength) with heat, and qualitatively it represents a transition to a more ductile behaviour.

When the free water cannot be drained out from the sample, the behaviour of the soil subjected to thermal loading is strongly affected. In consolidated undrained tests in which the volume variation is very small, the pore pressure increases due to the higher dilation of water compared to the soil skeleton. The pore pressure increase results in an effective stress decrease under constant external forces.

Experimental results on two clays reported by Hueckel and Pellegrini¹⁷ have shown the failure of samples heated from 70 to 90°C under undrained conditions with a constant stress deviator. This failure is due to the increase in pore pressure that causes an effective stress decrease under constant total stress conditions. The higher the initial deviatoric stress, faster the failure occurs.

3. THERMOVISCOPLASTIC CYCLIC CONSTITUTIVE MODEL

The thermoviscoplastic strain rate is defined in this model through a plastic flow rule. It is thus related to the yield surface, which depends on the stress state, temperature and internal variables.

The proposed thermoviscoplastic model is based on an existing isothermal elastoplastic multimechanism model with isotropic and kinematic hardening.¹⁸ Both deviatoric and isotropic yield surfaces are present in the model. A brief presentation of multimechanism elastoplasticity is given in the appendix. Readers may also refer to Pastor *et al.*¹⁹ for more information. The viscoplastic version of this model is easily obtained¹⁰ following Perzyna.²⁰

The formulation of the proposed thermoviscoplastic model is presented in the next section. As far as thermomechanical behaviour is concerned, this model is essentially based on the influence of temperature variation on the void ratio. This aspect is explicitly taken into account in the isotropic mechanism. The variation of the void ratio induces an evolution of the consolidation pressure. Hence, the deviatoric mechanisms will be affected by thermal loading, independently from the probable thermally-induced friction angle variation.

The total strain tensor ϵ due to thermomechanical loading is split in thermoelastic part ϵ^{Te} , and irreversible part ϵ^{Tp} .

Thermoelastic component: The temperature induces elastic as well as plastic strain. The thermoelastic part of the strain tensor ϵ^{Te} is considered as the superposition of a mechanical elastic strain under adiabatic conditions ϵ^e , and the reversible thermal strain ϵ^T . In incremental form we have

$$\dot{\epsilon}^{Te} = \dot{\epsilon}^e + \dot{\epsilon}^T \quad (1)$$

As in the isothermal version of the model, a non-linear elasticity with respect to mean effective pressure is assumed here. With p the mean effective pressure ($= \text{Tr}(\sigma')/3$) and σ_d the deviatoric effective stress tensor ($= \sigma' - \text{Tr}(\sigma')\mathbf{I}$), the mechanical elastic-strain part ϵ^e , is split in isotropic ($\epsilon_v^e = \text{Tr}(\epsilon^e)$) and deviatoric ($\epsilon_d^e = \epsilon^e - \frac{1}{3}\epsilon_v^e\mathbf{I}$) parts. The thermoelastic strain rate is then given by

$$\begin{aligned} \dot{\epsilon}_v^{Te} &= \frac{\dot{p}}{K} + \beta'_s \dot{T} \\ \dot{\epsilon}_d^e &= \frac{\dot{\sigma}_d}{G} \end{aligned} \quad (2)$$

with β'_s being the isotropic thermal expansion coefficient of the solid skeleton. It varies strongly with temperature and slightly with pressure.³ In the proposed model we have adopted the following relationship:

$$\beta'_s = (\beta'_{s0} + \zeta T) \zeta \quad (3)$$

in which ζ is the ratio between the critical state pressure for the initial state p_{c0} and the mean effective pressure p at ambient temperature:

$$\zeta = \frac{p_{c0}}{p} \quad (4)$$

and where β'_{s0} represents the isotropic thermal expansion coefficient at ambient or reference temperature T_0 and ζ corresponds to the slope of the variation of β'_s with respect to present temperature T at $\xi = 1$.

Equations (3) and (4) imply that the isotropic thermal expansion coefficient grows with the overconsolidation ratio and temperature.

The elastic moduli are given by

$$\begin{aligned} K &= K_{\text{ref}} \left(\frac{p}{p_{\text{ref}}} \right)^n \\ G &= G_{\text{ref}} \left(\frac{p}{p_{\text{ref}}} \right)^n \end{aligned} \quad (5)$$

where K_{ref} and G_{ref} are, respectively, the bulk and shear elastic moduli at a reference pressure p_{ref} (the value of mean effective pressure at which the elastic moduli are measured), and n is the non-linear elasticity exponent. Due to the lack of any significant information on the dependency of elastic moduli on the temperature in clays, here we have assumed that K and G are independent of the temperature. They vary only with the mean effective pressure. Meanwhile, the material may have a thermoplastic behaviour due to a change of temperature that induces a change of the mean effective pressure. In this case, we have a variation of K and G induced by the variation of the mean effective pressure caused by the temperature change.

At this point it should be recalled that here we have made the commonly admitted hypothesis that thermal elastic strain is isotropic. However, it could be also possible to make the isotropic thermal-stress assumption.

Remark. Such a non-linear elastic assumption is not thermodynamically admissible in the *elastic domain*, as the dependence on the second stress invariant is neglected. However, experimental results do not show a significant dependence of the elastic moduli on this former quantity. Probably, a coupled elastoplasticity (i.e. dependence of elastic moduli on hardening parameters) should be considered in establishing the thermodynamical basis in order to obtain this type of non-linear elasticity.

The thermoelasticity used in this model is mainly based on available experimental results. It is possible to derive a non-linear stress-dependent thermal expansion coefficient from a suitable choice of elastic energy but the dependence on the critical state pressure p_{c0} will introduce an elastoplastic coupling.

The form of equations (3) and (4) have been inferred from the experimental results performed on clays by ISMES.³ The reason for selecting such relationships may be partly understood from experimental results discussed later in the text (see also Figure 16).

Thermoviscoplastic strain rate component: The irreversible thermal effects are introduced by taking into account the dependence of yield surface, flow rule and internal variables on temperature. In the following sections the monotonous loading conditions are designated by the letter 'm' and the cyclic loading conditions by 'c'.

Deviatoric mechanisms

In order to take into account the anisotropy induced by the stress path, three deviatoric yield surfaces are written in three orthogonal planes (mechanisms k ; $k = \{1, 2, 3\}$) of the stress space (Figure 9). In each plane, a plane plastic strain hypothesis is assumed and a limit criterion very close to that of Mohr–Coulomb is obtained.

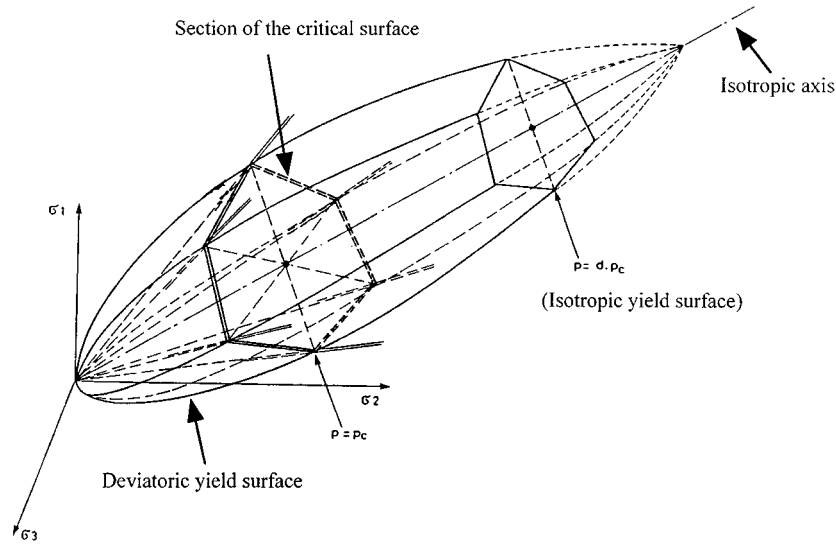


Figure 9. Presentation of yield surfaces (deviatoric and isotropic) in σ_I , σ_{II} , σ_{III} plane

Then the deviatoric monotonous yield surface for mechanism k is proposed as

$$f_k^m = q_k - p_k F_k r_k^m \sin \phi \quad (6)$$

where p_k and q_k are, respectively, the *reduced mean effective pressure* and *deviatoric stress* in the plane of each deviatoric mechanism k (see Figure 10) given as

$$q_k = \|\mathbf{s}_k\| = \frac{1}{2} \left[\frac{(\sigma'_{ii} - \sigma'_{jj})^2}{4} + \sigma'^2_{ij} \right]^{1/2} \quad k = \{3, 2, 1\}; i, j = \{1, 2\}, \{1, 3\}, \{2, 3\} \quad (7)$$

$$p_k = \frac{(\sigma'_{ii} + \sigma'_{jj})}{2}$$

with the vector $\mathbf{s}_k = (\frac{1}{2}(\sigma'_{ii} - \sigma'_{jj}), \sigma'_{ij})_k$. $\|\cdot\|$ denotes the norm of the vector. We note that σ'_{ij} is a component of the tensor σ' , and not the tensor itself (indicial notation is not used in this paper).

The function F_k is introduced to take into account the volumetric hardening or softening with respect to the critical state of the material as

$$F_k = 1 - b \left(\text{Ln} \frac{P_k}{p_{c0}} - \beta \varepsilon_v^p \right) \quad (8)$$

with b a numerical parameter ($0 \leq b \leq 1$), β the plastic compressibility modulus and p_{c0} the critical state pressure for the initial state.

The internal variable r_k^m (also called the degree of mobilized friction) introduces the effect of shear hardening of the soil (see later equation (19)). It also permits the decomposition of the behaviour domain into elastic, hysteretic and mobilized domains introduced by other model parameters called r_{el} , r_{hys} and r_{mob} in appropriate literature.¹⁸

It is possible to normalize the monotonous yield surface as

$$\mathcal{F}_k^m = \frac{\|\mathbf{s}_k\|}{p_k F_k \sin \phi} - r_k^m \quad (9)$$

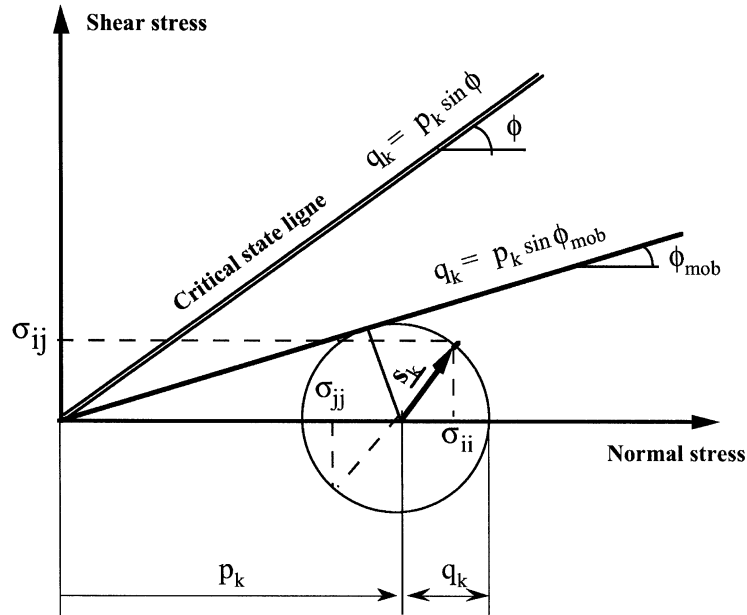


Figure 10. Stress state in the (i, j) plane of the deviatoric mechanism k ; $\sin \phi_{\text{mob}} = F_k r_k \sin \phi$

Hence, the normalized monotonous yield surface in the plane of the mechanism k is a circle with radius r_k^m , evolving during the loading.

Variables F_k and r_k both introduce isotropic hardening but have different evolution rules and different origins as well. The first one introduces the isotropic hardening associated with the plastic volumetric strain as in the Cam-Clay model. The second one represents the isotropic hardening caused by the deviatoric plastic strain in each mechanism. The mechanisms are coupled only due to the first one, as the second is attached to each physical plane representing the deviatoric mechanism.

ϕ is the friction angle at the critical state varying with the temperature as

$$\phi = \langle \phi_0 - gT \rangle \quad (10)$$

where ϕ_0 is the value of the friction angle at the ambient temperature and g is an average slope of variation of the friction angle with temperature, which can be either positive or negative. The symbol $\langle \cdot \rangle$ designates the Macaulay brackets: $[\langle x \rangle = x \text{ if } x \geq 0; \langle x \rangle = 0 \text{ otherwise}]$.

Obviously, the value of g and ϕ_0 obtained from experiments always result in positive values for the friction angle in the considered range of temperature (i.e. which do not introduce any phase change). However, as has been indicated in the introduction, experimental results (References 2 and 5) show that the friction angle may decrease or increase with the temperature.

In the same way the cyclic yield surface can be written as

$$f_k^c = q_k^c - p_k F_k r_k^c \sin \phi \quad (11)$$

with

$$q_k^c = \| \mathbf{s}_k^c \| = \| \mathbf{s}_k - p_k F_k \sin \phi (\mathbf{D}_k - \mathbf{n}_k r_k^c) \| \quad (12)$$

where \mathbf{D}_k is the normalized stress vector at the unloading (or reloading) point, and \mathbf{n}_k the exterior normal vector with respect to the active circle.^{10,18}

Thermoplasticity/thermoviscoplasticity

The yield surface presented in the last section may be included in the framework of elastoplasticity or viscoplasticity. The plastic flow rule is derived for each mechanism k and, in the first case the plastic strain rate becomes

$$\begin{aligned} (\dot{e}_v^{\text{Tp}})_k &= (\dot{e}_{ii}^{\text{Tp}} + \dot{e}_{jj}^{\text{Tp}})_k = \dot{\lambda} \cdot (\Psi_v)_k \\ (\dot{e}_d^{\text{Tp}})_k &= ((\dot{e}_{ii}^{\text{Tp}} - \dot{e}_{jj}^{\text{Tp}}), \dot{e}_{ij}^{\text{Tp}})_k = \dot{\lambda} \cdot (\Psi_d)_k \end{aligned} \quad k = \{3, 2, 1\}, \quad i, j = \{1, 2\}, \{1, 3\}, \{2, 3\} \quad (13)$$

The direction of plastic flow in each mechanism is obtained by assuming the normality rule in the deviatoric plane together with Roscoe's dilatancy rule. $(\Psi_v)_k$ and $(\Psi_d)_k$ are thus defined by

$$\begin{aligned} (\Psi_v)_k &= -\alpha_k^{\Xi} \left(\sin \psi - \frac{[\mathbf{s}_k \cdot (\Psi_d)_k]}{p_k} \right) \\ (\Psi_d)_k &= \partial_s f_k^{\Xi} \end{aligned} \quad \Xi = \text{m or c} \quad (14)$$

where ψ is the dilatancy angle corresponding to the characteristic line. The plastic multiplier $\dot{\lambda}$ is evaluated from the persistency (or consistency) condition; i.e. $\dot{\lambda} \dot{f} = 0$, after introducing the following evolution law for the internal variable r_k^{Ξ} :

$$\dot{r}_k^{\Xi} = \dot{\lambda} l^{\Xi} \quad \Xi = \text{m or c} \quad (15)$$

with

$$l^{\text{m}} = \frac{(1 - r_k^{\text{m}})^2}{a} \quad \text{and} \quad l^{\text{c}} = \frac{(1 - r_k^{\text{c}})^2}{a} \cdot \frac{\|\mathbf{s}_k^{\text{c}}\|}{\|\mathbf{s}_k^{\text{c}}\| - \mathbf{n}_k \cdot \mathbf{s}_k^{\text{c}}}$$

a is a parameter defined as

$$a = a_2 + \alpha_k^{\Xi} (a_1 - a_2) \quad (16)$$

where a_1 and a_2 are numerical parameters. One can refer to Reference 18 for further information on parameter α_k^{Ξ} which may vary with r_k^{Ξ} and adjusts the dilatancy during shearing. The role of a_1 and a_2 can be understood by taking α_k^{Ξ} equal to unity. In that case, the internal variable r_k^{Ξ} represents a hyperbole with a slope at origin equal to $1/a_1$.

Remark. In equation (14) ψ appears instead of ϕ . The former is known as the characteristic angle and the later as the friction angle at the critical state. The difference between the characteristic and critical state is a much debated question. The later describes the perfect plasticity state while the former represents the transition from the contracting to the dilating behaviour under compression. The discussion about this topic is out of the scope of this paper. In the proposed model we have kept the structure of the original elastoplastic model in which the possibility of different values for the friction angle in the expression of the yield surface and the dilatancy rule were included. By choosing the same value for ϕ and ψ , the standard critical state approach may be recovered.

The rate-dependent (thermoviscoplastic) constitutive model is a Perzyna-type viscoplastic model. Then, the following relationships may be used for evaluating the irreversible strain for

each mechanism k :

$$\begin{aligned} (\dot{\epsilon}_v^{\text{Tp}})_k &= \frac{\left\langle \frac{f_k^\Xi}{f_0} \right\rangle^{n_\mu}}{\mu} \cdot (\Psi_v)_k \\ (\dot{\epsilon}_d^{\text{Tp}})_k &= \frac{\left\langle \frac{f_k^\Xi}{f_0} \right\rangle^{n_\mu}}{\mu} \cdot (\Psi_d)_k \end{aligned} \quad \Xi = m \text{ or } c \quad (17)$$

where μ is a measure of viscosity which may vary with the temperature. One would conclude that as μ tends to zero, the rate-independent (thermoplastic) constitutive model is recovered. f_0 is a reference stress introduced for dimensional requirements (here taken equal to 1 MPa). The direction of plastic flow in each mechanism is the same as in equation (14).

The following viscosity law may be adopted by analogy with the temperature-dependence of water's viscosity:

$$\mu = \eta e^{\zeta/T} \quad (18)$$

where η and ζ are physical constants. n_μ is a material viscosity parameter. The choice of the viscosity law, same as water may be done in first approximation as a plausible hypothesis in absence of data on such a law for the studied materials. In general, to obtain the model parameters (remembering that the elastoplasticity is an asymptotic state for such a viscoplastic model), the best strategy is to perform tests with very low loading rate in order to first determine the 'elastoplastic'-type parameters (rate-independent) and then higher loading rates for viscosity-type parameters. These later may be replaced by creep-type tests.²¹ The viscosity exponent n_μ may be determined using creep tests with several loading steps. In all numerical applications presented in this paper, n_μ has been assumed equal to *unity* and μ has been supposed to be *constant*. Without specific tests it is not possible to determine these quantities.

The evolution rule for the degree of mobilized friction is given by the following relationship:

$$\dot{r}_k^m = \frac{(1 - r_k^m)^2}{a} \frac{\left\langle \frac{f_k^m}{f_0} \right\rangle^{n_\mu}}{\mu} \quad \text{and} \quad \dot{r}_k^c = \frac{(1 - r_k^c)^2}{a} \frac{\left\langle \frac{f_k^c}{f_0} \right\rangle^{n_\mu}}{\mu} \frac{\|\mathbf{s}_k^c\|}{\|\mathbf{s}_k^c\| - \mathbf{n}_k \cdot \mathbf{s}_k^c} \quad (19)$$

Isotropic mechanism

With only deviatoric mechanisms, no irrecoverable strain is developed under isotropic loading conditions. Thus an isotropic mechanism is necessary for taking into account such effects. For thermal loading conditions this mechanism plays a very crucial role. The isotropic yield surface is proposed as

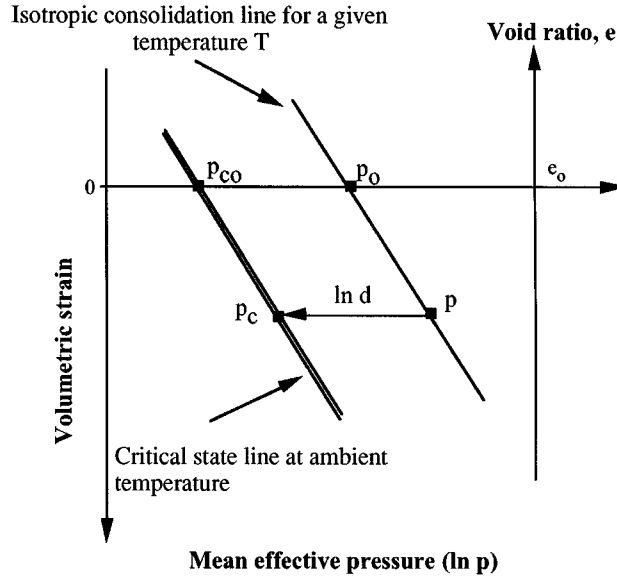
$$f_{\text{iso}}^\Xi = |\pi^\Xi| - dp_{c0} e^{\beta \epsilon_v^{\text{Tp}}} r_{\text{iso}}^\Xi \quad \Xi = m \text{ or } c \quad (20)$$

with $\pi^m = p$, $\pi^c = p - (D_{\text{iso}} - n_{\text{iso}} r_{\text{iso}}^c) dp_{c0} e^{\beta \epsilon_v^{\text{Tp}}}$. r_{iso}^Ξ is the degree of mobilization of this mechanism ($0 \leq r_{\text{iso}}^\Xi \leq 1$). D_{iso} is the normalized mean effective pressure at the unloading (or reloading) point, and n_{iso} ($= -1$ or $+1$) indicates the direction of the previous loading.^{10,18}

In 'e-Ln(p)' plane, d represents the distance between the critical state line at ambient temperature and the isotropic consolidation curve for a given temperature (Figure 11).

As the isotropic consolidation line becomes closer to the critical state line with temperature increasing,¹¹ the next relationship is assumed for modelling this phenomenon:

$$d = d_0 e^{-\beta x} \quad (21)$$

Figure 11. Definition of parameter d

χ is the variation of the void ratio with temperature and is determined as

$$\chi = \frac{T - T_0}{a_t(T - T_0) + b_t} \quad (22)$$

a_t and b_t being numerical parameters determined from experimental results.

The expression of d implicitly assumes that the volume variation due to thermal expansion (reversible part) is negligible compared to that caused by thermoplasticity (irreversible part) resulting in the compaction of nearly normally consolidated clays. This assumption may be verified using usual values for thermal expansion coefficients and observed settlement in heated samples. Moreover, the variation of thermally-induced volumetric plastic strain follows a hyperbolic evolution with the temperature increase for normally consolidated clays. This may be easily verified from equations (20)–(22) in the case of thermal loading under constant isotropic stress. Parameters a_t and b_t may be directly estimated if such tests are available.

The isotropic mechanism is assumed to be associated and the plastic strain rate can be calculated as

$$(\dot{\epsilon}_v^{Tp})_{iso} = \dot{\lambda}(\Psi_v)_{iso} \quad (23)$$

for the thermoplastic formulation, and by

$$(\dot{\epsilon}_v^{Tvp})_{iso} = \frac{\left\langle \frac{f_{iso}^{\Xi}}{f_0} \right\rangle^{n_p}}{\mu} (\Psi_v)_{iso} \quad \Xi = m \text{ or } c \quad (24)$$

for rate-dependent constitutive model.

In both formulations an associated flow rule is assumed. The thermoviscoplastic strain increment is thus proportional to

$$(\Psi_v)_{\text{iso}} = \frac{1}{3} \text{sgn}(\pi^{\bar{\epsilon}}) \quad \Xi = m \text{ or } c \quad (25)$$

The coupling between deviatoric and isotropic mechanisms by the volumetric hardening parameter (volumetric plastic strain) results in an expansion of deviatoric yield surfaces during thermal loading. On the other hand, there may be a decrease in the value of the friction angle (equation (10)), which can counterbalance the first effect. These aspects will be discussed in the next section.

Remark. In the rate-independent formulation, one could verify that as all four mechanisms are coupled, the determination of the plastic multiplier from the consistency condition needs a system of equations to be solved. In the thermoviscoplastic formulation, there is no need for such heavy computations but additional model parameters have to be taken into account.

4. MODEL RESPONSE

In this section we shall discuss the material response and the evolution of the stress point due to thermal loading in drained and undrained conditions with respect to different mechanisms.

Isotropic yield surface

Experimental results¹ show that in a normally consolidated clay under drained conditions and at constant effective mean pressure the temperature increase results in an irreversible compaction. To model such a behaviour, the isotropic yield surface is assumed to underlie the evolution presented in Figure 12 (Normally consolidated state): The test is started at point (0) in which the stress point P_0 is on the yield surface f_0 (equation (20)). With a temperature increase ($\Delta T = T_1 - T_0$), d decreases (equation (21)) resulting in a shrinkage of the isotropic yield surface (1). With elastoplastic assumption, an instantaneous volumetric plastic strain is generated (isotropic hardening) in order to maintain the yield surface f_1 on the stress point. In viscoplastic formulation, the yield surface will gradually regain the stress point with the generation of the volumetric plastic strain modulated by the viscosity of clay.

For overconsolidated clays (Figure 12—Overconsolidated state), the stress point P_0 is initially inside the yield surface f_0 due to the high value of the critical state pressure for the initial state P_{c0} . During heating and under drained conditions and isotropic stress state, a dilational thermoelastic strain is generated (equation (2)). Moreover, d decreases conforming to equation (21) resulting in shrinkage of the isotropic yield surface (1). If the shrinkage is insufficient, the obtained thermal strain is reversible. Otherwise, the model will behave as in the case of normally consolidated clays, i.e. thermoplastic compaction.

Deviatoric yield surface

The evolution of deviatoric yield surfaces follows two opposite phenomena: a shear-resistance decrease caused by variation of the friction angle ϕ and a thermal compaction due to volumetric thermoplastic strain increasing the shear resistance. Following the initial state and the loading history, each of these aspects will affect the response. The model's deviatoric behaviour will essentially depend on the initial density (p_{c0}), the volumetric plastic strain and the friction angle.

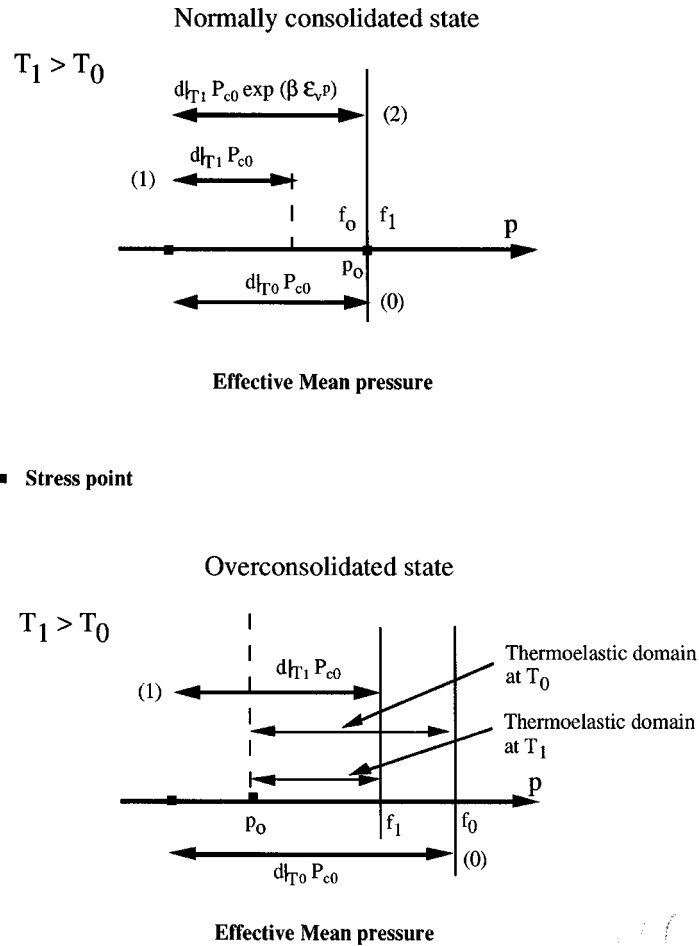


Figure 12. Evolution of the normalized isotropic yield surface with temperature

Assuming a decrease of the friction angle at critical state (ϕ) with temperature (equation (12)) results in a decrease of the deviatoric resistance (equation (6)). In elastoplastic formulation, the stress point has to remain constantly on the yield surface, thus instantaneous deviatoric plastic strain is generated. During cooling, the deviatoric yield surfaces expand due to the increase of friction angle and the material behaves almost like a thermoelastic material with reversible strain.

The coupling of deviatoric mechanisms to the isotropic mechanism (via the volumetric plastic strain) introduces a densification tendency that enlarges the deviatoric yield surfaces and slows down thermoplasticity (equation (6) and (8)). Therefore, it operates in the opposite direction to the preceding case. The combination of these two aspects governs the response of the model in the simulation of the clay's deviatoric behaviour subjected to heating.

Finally, in the thermoviscoplastic formulation the variation of the viscosity with temperature may largely affect the amplitude of thermally induced strain.

Thermomechanical behaviour under undrained conditions

The role of the free water is taken into account by conservation laws for saturated poro-elastoplastic materials subjected to thermal loading. The mass conservation equation in homogeneous undrained conditions, for which there is no relative displacement of the fluid with respect to the solid skeleton, is given as

$$\theta \beta'_f \dot{p}_f = -\dot{\varepsilon}_v + [\theta \beta'_f + (1 - \theta)(\beta'_{s0} + \zeta T)] \dot{T} \quad (26)$$

where θ is the effective porosity of the solid skeleton relative to the volume occupied by the free water only, and p_f and β'_f are, respectively, the pore pressure and the compressibility of free water.

Therefore, the variation of the free-water pore pressure is attributed to two factors: the variation of skeleton volume (compaction or dilation) with respect to its effective behaviour and the thermal dilation of free water. The thermal expansion coefficient of water depends on temperature and in some way on pressure.

5. VALIDATION OF THE CONSTITUTIVE MODEL

Validation of the proposed cyclic thermo-(visco)plastic model is carried out and its capacity to predict the major aspects of clay behaviour using various experimental tests under different coupled thermomechanical loading paths is examined in Reference 6. Here we present two examples, the first one concerns the Boom Clay and the second one the Pontida Clay.

Validation test of the model for Boom Clay

The numerical simulations were compared with experimental results obtained on the Boom Clay. For these numerical simulations the parameters of the constitutive model have been identified using undrained triaxial²² and consolidation³ tests under isothermal conditions at ambient temperature. The experimental results used for determination of the isothermal model parameters are given in Figures 13 and 14. The first one represents numerical and experimental

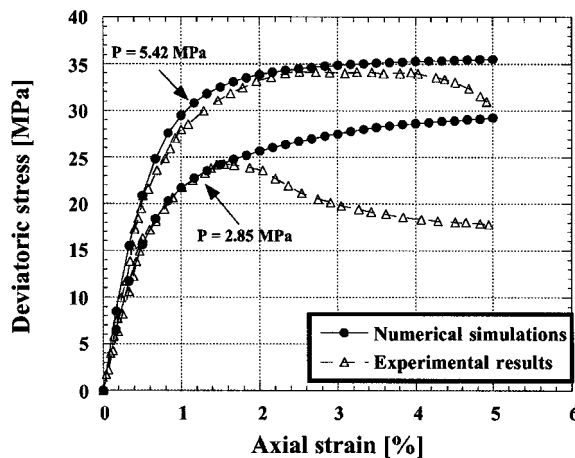


Figure 13. Undrained triaxial tests on Boom clay with confining pressure 2.85 and 5.42 MPa (experimental data from Horseman *et al.*²⁴)

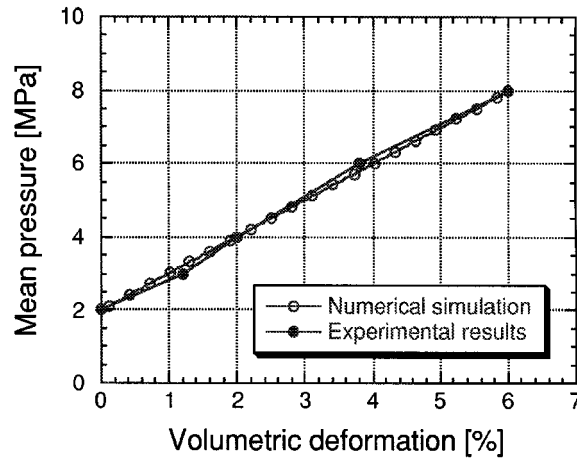


Figure 14. Consolidation tests on Boom Clay (experimental data from Baldi *et al.*³)

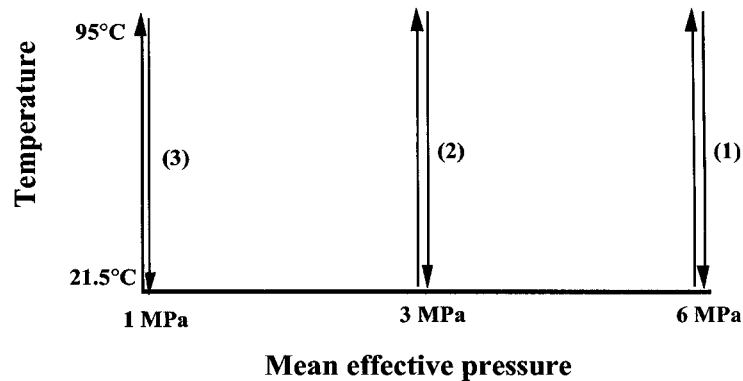


Figure 15. Thermal loading for the validation test

results in axial strain–deviatoric stress plane for two values of initial cell pressures (2.85 and 5.42 MPa). To determine thermomechanical parameters, the experimental results of thermal loading from ambient temperature of 21.5°C to high temperature of 95°C at two different initial stress states were used³ (cases 1 and 3 in Figure 16). Case 2 was used for comparison. The test consists of applying a thermal load (Figure 15) to a normally consolidated (case 1), a slightly overconsolidated (case 2) and an overconsolidated sample (Case 3). Cases 1 and 3 were used to determine the thermal parameters of the model with the assumption that in case 3 the behaviour is thermoelastic. Therefore, only case 2 corresponds to a numerical prediction. The parameters of the model are presented in Table I. The comparison between numerical prediction and experimental results is satisfactory during thermal loading and unloading.

Figure 17 represents an isotropic thermomechanical loading path, used for validation of the constitutive model with parameters determined from previous path. The experimental results have been taken from Baldi *et al.*²³

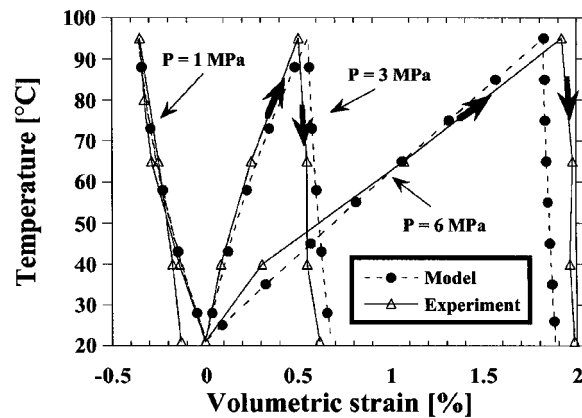


Figure 16. Comparison of experimental results (from Baldi *et al.*³) and numerical predictions with the proposed model. Thermal loading/unloading under constant isotropic stress

Table I. Model parameters for Boom Clay

Elastic (equation (5))	$K_{\text{ref}} = 150 \text{ MPa}$, $G_{\text{ref}} = 130 \text{ MPa}$, $n = 0.4$
Plastic (equation (8))	$\phi_0 = 19.5^\circ$, $\beta = 14$, $p_{c0} = 6 \text{ MPa}$
Dilatancy (equation (14))	$\psi = 23^\circ$
Hardening (equations (8), (16), (21))	$a_1 = a_2 = 0.002$, $b = 0.6$, $d_0 = 1.3$
Domains (equation (9))	$r_{\text{el}} = 0.001$, $r_{\text{hys}} = 0.01$, $r_{\text{mob}} = 0.1$
Viscosity (equation (17))	$\mu = 10^9 \text{ Pa mn}$, $n_\mu = 1$
Thermal (equations (3), (10), (22))	$\beta'_s = 1.3 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, $g = 0.0085$, $a_t = 0.5$, $b_t = 3350$

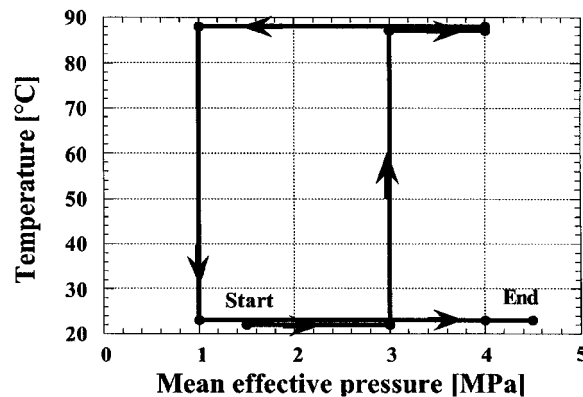


Figure 17. Thermomechanical loading paths

Numerical simulations and experimental results giving the variation of volumetric strains with the temperature are shown in Figures 18 and 19, respectively. It should be noted here that the first part of the loading path (isotropic consolidation from 1.5 to 3 MPa and first thermal loading from 22 to 88°C) has been used for calibrating model parameters. Figure 18 shows that

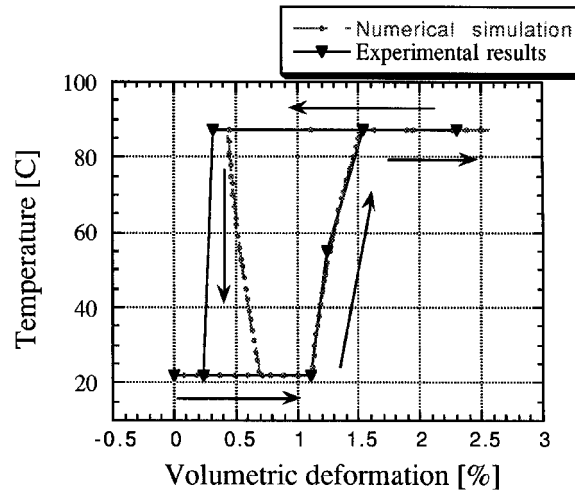


Figure 18. Comparison between experimental results (from Baldi *et al.*²³) and numerical predictions in the volumetric-strain/temperature plane (loading paths of Figure 17)

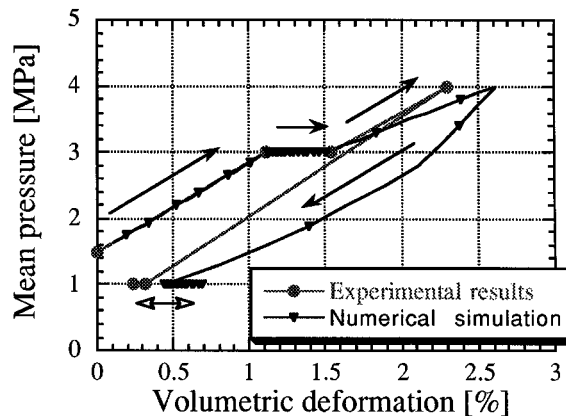


Figure 19. Comparison between experimental results (from Baldi *et al.*²³) and numerical predictions in the volumetric-strain/mean effective pressure plane (loading paths of Figure 17)

predictions are quite satisfactory in the volumetric strain/temperature plane. The only difference is during thermal unloading in the overconsolidated state, where the model predicts a dilation as opposed to the experiment. In the volumetric-strain/mean-pressure plane (Figure 19), the experimental results show that the apparent volumetric stiffness is independent of temperature. This is not the case in numerical predictions. However, in general the numerical predictions in this isotropic case remain even quantitatively acceptable. The selected example shows the complexity of thermomechanical loading and confirms the suitability of the proposed model.

Validation of the model for Pontida Clay

The validation tests were also conducted on remoulded Pontida Clay, for which triaxial deviatoric thermal tests are available.³ These tests were made at two different temperatures for

two different over-consolidation ratios (OCR = 5 and 12.5). The experimental results show that the temperature has caused a shear resistance decrease and a decrease of dilatancy.

The model parameters have been determined (Table II) using these isothermal experiments (Fig. 20). The results of numerical simulations for thermomechanical tests conducted at 95°C are quite satisfactory for the stress deviator as well as the volumetric strain (Fig. 21). They show the capacity of the model to reproduce the major aspects of the influence of temperature on mechanical behaviour.

Table II. Model parameters for Pontida Clay

Elastic (equation (5))	$K_{\text{ref}} = 200 \text{ MPa}$, $G_{\text{ref}} = 150 \text{ MPa}$, $n = 0.6$
Plastic (equation (8))	$\phi_0 = 35^\circ$, $\beta = 51$, 5 , $p_{c0} = 1.1 \text{ MPa}$
Dilatancy (equation (14))	$\psi = 24.5^\circ$
Hardening (equations (8), (16), (21))	$a_1 = a_2 = 0.004$, $b = 0.3$, $d_0 = 1.2$
Domains (equation (9))	$r_{\text{el}} = 0.0001$, $r_{\text{hys}} = 0.001$, $r_{\text{mob}} = 0.01$
Viscosity (equation (17))	$\mu = 10^9 \text{ Pa mn}$, $n_\mu = 1$
Thermal (equations (3), (10), (22))	$\beta'_s = 1.3 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, $g = 0.006$, $a_t = 0.16$, $b_t = 1$

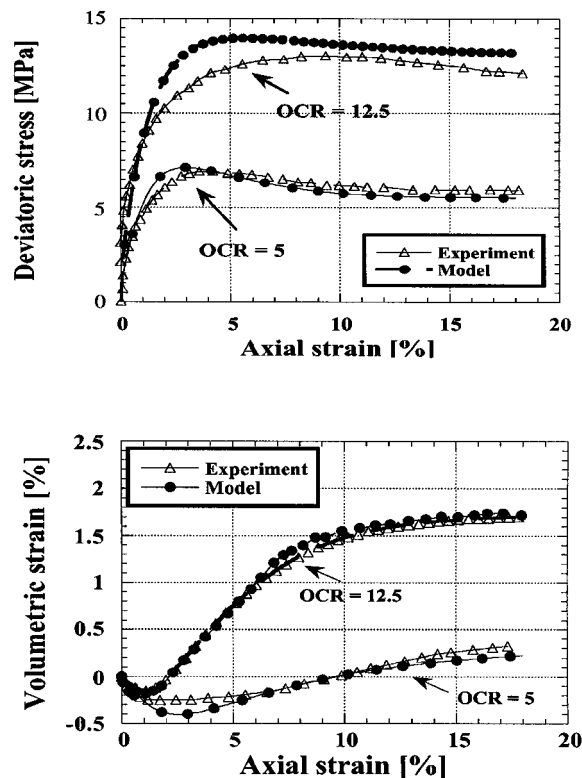


Figure 20. Drained triaxial results (from Baldi *et al.*³) and numerical simulations at 20°C for two OCR values

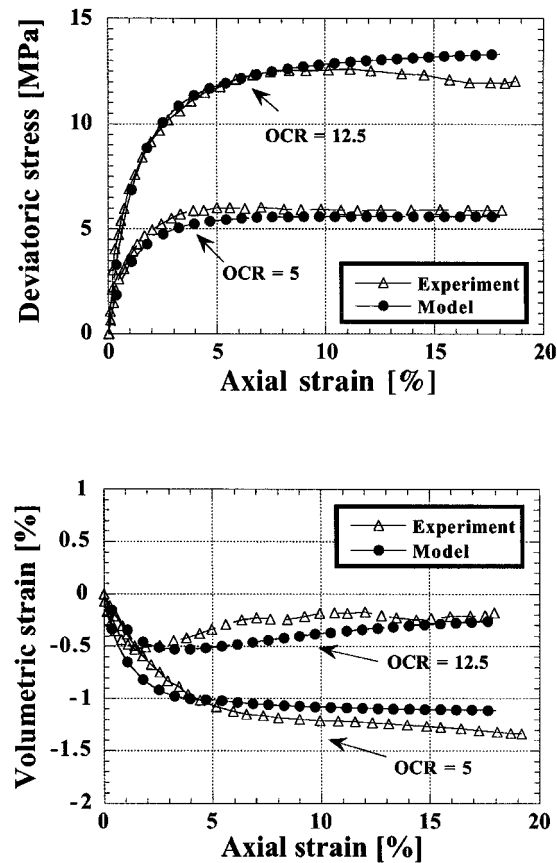


Figure 21. Comparison of experimental results (from Baldi *et al.*³) and numerical simulations at 95°C for two OCR values

6. CONCLUSIONS

A thermoviscoplastic cyclic model is proposed for clays subjected to thermomechanical loading on the basis of available experimental results. This model is a generalization of a pre-existing elasto(visco)plastic model developed for such materials under isothermal conditions. Its multimechanism structure facilitates the formulation of the proposed constitutive model. The dependence on temperature of yield surfaces and hardening parameters as well as other constitutive and physical parameters are discussed. The continuous variation of mechanical characteristics with temperature results in a transition to a more ductile behaviour with a loss of shear resistance in some cases. This later depends on the volume variations due to the temperature increase. If during thermal loading the material contracts, the density and shear resistance increase. The volume variations with temperature depend on the overconsolidation state. The transition from thermoelastic dilational to thermoplastic contracting behaviour is correlated with the stress state. The role of adsorbed water is implicitly included in the constitutive model, while the free water is analysed through conservation equations for saturated poro-elasto-plastic materials.⁶ The mass conservation equation gives the evaluation of the pore-pressure generated during undrained conditions. Under constant external forces, the pore-pressure generation results in a decrease of

effective stress. The model gives satisfactory results under various loading paths for the studied cases. More specifically, we note that the irreversible contracting thermal strain is correctly estimated. In comparison with other Cam-Clay-based thermo-plastic models, this model seems to be more efficient in reproducing experimental results. It has been introduced in a finite element code in order to solve initial-boundary value problems under complex hydro-thermomechanical loading conditions.⁶

Meanwhile more studies are needed for detecting the probable shortcomings of the model. There is still a lack of well-controlled laboratory and *in situ* experiments on different clays. However, several industrial projects related to environmental and waste-disposal programmes open new ways of investigation in this domain. The extension of the proposed model for the case of unsaturated clays has been recently investigated²⁴ and is still under study.

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APPENDIX: MULTIMECHANISM ELASTOPLASTICITY

The proposed thermoviscoplastic model is based on an existing isothermal orthotropic elasto(visco)plastic multimechanism model with isotropic and kinematic hardening.^{10,18,25} It is a generalization of the theory of critical state²⁶ in which the theory of coupled multimechanism elastoplasticity as in the Koiter–Mandel theory of elastoplasticity²⁷ is introduced. In each plane, a plane plastic strain hypothesis is adopted. The limit criterion is very close to that of Mohr–Coulomb. The orthotropic nature of the model stipulates that the physical co-ordinate axes are parallel to principal stress directions. Then the model is based on the representation of all irreversible phenomena by four coupled elementary elastoplastic mechanisms: three orthogonal deviatoric and one isotropic. These four mechanisms are activated during primary as well as cyclic loading. Two kinds of internal parameters are needed; some subjected to a continuous flow rule and others obeying a discontinuous evolution so that a double memory is built in the model. The second type of internal parameters are sometimes referred to as discrete-memory internal parameters as they represent only the last loading reversal in each plane. Each deviatoric mechanism has its own hardening parameters related to distortion in the corresponding plane, and all four mechanisms are coupled by the isotropic hardening parameter ε_v^p (volumetric plastic strain = $\sum_{k=1}^4 (e_v^p)_k$). If the principal stress directions vary under loading conditions, then normals to each deviatoric plane vary in the stress space and become additional internal parameters. This formulation of the multimechanism model has been developed in isothermal conditions but has not been generalized to thermoplastic models for clays in which an orthotropic assumption seems to be justified. Finally, in all versions the viscoplastic implementation is easily obtained following Perzyna.²⁰

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